

### 3. Complete Contracts II: Static Multilateral Contracting

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#### 3.4 Collusion

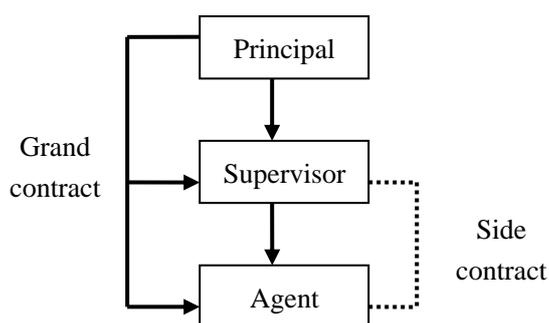
##### 3.4.1 Background

###### Collusion

Collusion is a prevalent phenomenon in organizations, as well even in classes and families. Collusion can be cataloged to two kinds: (lateral) collusion in market and (vertical) collusion within organization. The former belongs to industrial organization theory, so we will focus on the latter. For instance, the collusion between the union and management in labor economics, the board of directors and management in financial economics, officials and constituency in public economics, government agencies and industry or interest group in regulatory economics, and agents in contract theory (e.g., auctions and moral hazard in teams, tournament).

As Laffont said, collusion is the most important characteristic of organization. According to Stigler (1971), three factors that determinate the formation of collusion are: mobilization costs (which depends on the nature of the interest group), transfer costs (which depends on the forms of reciprocity and supervision technology), information structure.

A typical process of vertical collusion is the following figure (Tirole, 1992):



###### Two approaches: enforceability versus self-enforceability

The technology for side transfers differs in specific situations. Sometimes, the cost  $s$  of side transfer for the donor is smaller than the value  $\tilde{s}$  for the recipient, i.e.  $s > \tilde{s}$ , we call that is collusion; if  $s < \tilde{s}$ , we call it cooperation (It is not true! It depends on the law, see Nie (2013)). However, more elusive is the issue of how side contracts are enforced. According to the enforceability of side contract, Tirole (1992) divides all the existing literatures into two parts: enforceable-side-contracts approach and self-enforceable-side-contracts.

Next, we summarize the comparison as follows:

	<b>Enforceability</b>	<b>Emphases</b>	<b>Enforcements</b>	<b>Techniques</b>
<b>Enforceability approach</b>	Exogenous	The proof of collusion	NO	Classic contract theory
<b>Self-enforceability approach</b>	Endogenous	The foundation of side contracts	Word of honor, revenge, and reputation	Dynamic mechanism design

### 3.4.2 Basic insights

**Theme 1** (collusion proofness): Under some conditions, there is no loss in designing organizations while do not leave scope for collusion.

Here, side transfers do not occur. But it only predicts the final outcome but rather that the contracting structure (collusion-free grand contract or delegation) that lead to this outcome. In fact, if S is risk-neutral and has symmetrical information with A, letting S the residual claimant will not lead to (extra) cost of collusion-proof (for moral hazard problem) (Tirole, 1986).

**Theme 2** (equilibrium collusion): in other circumstances, letting members of the organization collude is optimal.

Side transfers may be good for the organization if they are acts of cooperation (relatedly, direct transfers among members may be less costly than transfers channeled through a center). Also, collusion proof may be not good for the organization, but it is too costly to fight.

**Theme 3** (lack of discretion and bureaucratic behavior): The fear of collusion leads organizations to reduce stakes and to make less use of decentralized information.

Reducing the sensitivity of decisions to decentralized information, lowers the gains from collusion.

**Theme 4** (linkage of incentives): Standard sufficient statistics principles for rewarding agents do not hold in the presence of collusion.

If  $g(y, a) = h_i(y, a_{-i})P_i(T(y), a)$ , we say a function  $T(y)$  is sufficient statistics of  $y$  respect to  $a$ . Because agents are linked through potential side contracting, sufficient statistics results must be extended to the level of the group. The point is that individual rewards that are less sensitive to individual performance and depend on team performance make it less likely that the supervisor will collude with particular team members to manipulate reports about individual performances. For example, the “linking punishment” in ancient China.

### 3.4.3 A Model

#### Information

Following Laffont-Tirole (1991), suppose that there are three parties: a principal (P, regulator), a supervisor (S, auditor) and an agent (A, utility firm). Production  $x$  is 0 or 1 (when  $x = 1$ , the gross surplus to P is  $V$ ). The marginal cost  $\beta$  is private information for A, and takes value  $\underline{\beta}$  with probability  $\nu$  and  $\bar{\beta}$  with  $1 - \nu$ .

**Utility**

P and A are both risk neutral. A's utility is  $U = w - \beta x$  and his reservation utility is 0. In the absence of supervisor, P's welfare is  $W = Vx - w + \alpha_A U$ . There are two alternative justifications for this assumption:  $W$  is a social welfare function; or A is risk averse, and shares some risk with P.

**No supervisor**

Without a supervisor, the model is a model of classical monopoly pricing or pure adverse selection. The principal's question is to design optimal wage. P will set

$$w = \bar{\beta} \text{ if}$$

$$V - \bar{\beta} + \alpha_A v \Delta \beta \geq v(V - \underline{\beta})$$

or

$$v \leq \frac{V - \bar{\beta}}{V - \underline{\beta} - \alpha_A \Delta \beta} \quad (10-1)$$

and

$$w = \underline{\beta} \text{ if}$$

$$v > \frac{V - \bar{\beta}}{V - \underline{\beta} - \alpha_A \Delta \beta} \quad (10-2)$$

*Proof:*

Consider first P give all kinds of A payment  $\bar{\beta}$  which must satisfy A's IR constraint. And P's revenue is

$$\begin{aligned} W &= v(V - \bar{\beta} + \alpha_A U_{\bar{\beta}}) + (1-v)(V - \bar{\beta} + \alpha_A U_{\bar{\beta}}) \\ &= V - \bar{\beta} + \alpha_A v \Delta \beta \end{aligned}$$

Giving  $\underline{\beta}$ , only low-cost type A will participate the game, so P's revenue is

$$W = v(V - \underline{\beta} + \alpha_A U_{\underline{\beta}}) = v(V - \underline{\beta}) \quad \mathbf{Q.E.D.}$$

*Think*

☛ Why P can't choose the quantity as control variable?

**Supervisor**

In order to discuss price discrimination or collusion, we will assume that (10-1) holds. We introduce a supervisor (S), he learns a signal  $\sigma \in \{\underline{\beta}, \phi\}$ . If  $\beta = \underline{\beta}$ , S learns it with probability  $\zeta$  and nothing with  $1 - \zeta$ . If  $\beta = \bar{\beta}$ , S learns nothing (while his signal is useless). We assume that the signal is **hard** information and there is symmetric information between S and A. S's

report  $r \in \{\sigma, \phi\}$ . S's utility is  $S(s) = s$ , and his reservation utility is 0 (his cost is 0, see Tirole 1986). P's welfare is  $W = Vx - w - s + \alpha_A U + \alpha_S S$ .

### Timing

(1) A privately learns  $\beta$ , and S generates  $\sigma$ , which means agent has *interim* participation constraints; (2) P offers S and A a grand contract  $\{w, s, x\}$ ; (3) S makes a take-it-or-leave-it side contract to A (if any); (4) contracts are implemented.

### No collusion

S is given a flat income  $s \equiv 0$  and reports truthfully. If  $r = \underline{\beta}$ , then  $w = \underline{\beta}$  and A has 0 rent. If  $r = \phi$ , P's posterior belief about  $\beta$  is

$$\Pr ob(\underline{\beta} | \phi) \equiv \hat{\nu} = \frac{\nu(1-\zeta)}{\nu(1-\zeta) + (1-\nu) \times 1} < \nu$$

So, together with (10-1), we know that it is optimal to offer  $w = \bar{\beta}$ . Welfare is therefore,

$$W^d = \nu\zeta(V - \underline{\beta}) + \nu(1-\zeta)(V - \bar{\beta} + \alpha_A \Delta\beta) + (1-\nu)(V - \bar{\beta}) \quad (10-3)$$

where, "d" stands for "price discrimination". Under no discrimination or uniform pricing,

$$W^u = V - \bar{\beta} + \alpha_A \nu \Delta\beta \quad (10-4)$$

Clearly,  $W^d > W^u$ , which means that it is beneficial for P to introduce a supervisor.

### Collusion and policies

We now allow S and A to collude, and transaction efficiency is  $k \in [0, 1]$ . That is, if A transfers  $t$ , S receives  $kt$ . When  $k = 0$ , it is no-collusion case. When do S and A collude? If  $\sigma = \phi$ , collusion does not occur. Only if  $\sigma = \underline{\beta}$ , S and A have incentive to collude for the information rent  $\Delta\beta$ . In order to induce S to reveal true information when  $\sigma = \underline{\beta}$ , P must pay S  $s = k\Delta\beta$  only when S reports  $\sigma = \underline{\beta}$ . P's welfare:

$$\begin{aligned} \tilde{W}^d &= \nu\zeta[V - \underline{\beta} - (1-\alpha_S)k\Delta\beta] + \nu(1-\zeta)[V - \bar{\beta} + \alpha_A \Delta\beta] + (1-\nu)(V - \bar{\beta}) \\ &= W^d - \nu\zeta(1-\alpha_S)k\Delta\beta \end{aligned} \quad (10-5)$$

Note that: (1) A can get positive information rent when he is  $\underline{\beta}$  but S reports  $\sigma = \phi$ , so it is not a truth-telling mechanism; (2)  $\tilde{W}^d$  decreases with  $k$  and is smaller than  $W^d$  (it means

that collusion breaks loss to P). We call the policy the *incentive policy*. Theoretically P can sell the organization to risk-neutral S at a price equals to  $W^d - s_0$ , and S gets 0 rent.

In contrast, P can decide not to use S's information, then offers  $s \equiv 0$  and  $w = \bar{\beta}$  (according to 10-1). This yields welfare:

$$\tilde{W}^u = V - \bar{\beta} + \alpha_A v \Delta \beta = W^u \quad (10-6)$$

This latter policy can be interpreted as eliminating S's discretion, which is called the *bureaucratic policy*.

Note that:

$$(1) k = 0, \tilde{W}^d = W^d > \tilde{W}^u = W^u;$$

$$(2) \alpha_A > \alpha_S, k = 1, \tilde{W}^d < \tilde{W}^u;$$

$$(3) \alpha_A > \alpha_S, k \in (0, k^*), \tilde{W}^d > \tilde{W}^u.$$

These results correspond to Theme 1, 3 and 4.

*Remark 1*

The difficulty of discrimination illustrates the presence of collusion. The threat of collusion induces the widespread mechanism of low-powered scheme (cost-plus contracts in procurement or cost-of-service pricing in regulation, flat wages for employees). There is a Chinese saying, “不尚贤，使民不争”.

*Remark 2*

An alternative incentive for S to tell truth is the threat of being punished (equals to decreased wage) if caught colluding with A, once P randomly selects an outside auditor (Kofman-Lawarree, 1993).

### **Collusion proofness**

Furthermore, with participation constraints *ex ante*, we give a collusion-proof contract (truth-telling mechanism), which maximizes expected social welfare subject to agents' IR, AIC and SIC. Index the three states of nature in the following way: 1:  $\sigma = \underline{\beta}$ ; 2:  $\sigma = \phi$  and

$\beta = \underline{\beta}$ ; 3:  $\sigma = \phi$  and  $\beta = \bar{\beta}$ . A's and S's expected utilities are  $w_i$  and  $s_i$ . So, the program is (as 10-5):

$$\begin{aligned} & \underset{\{s_i, w_i, x\}}{\text{Max}} \nu \zeta (V - w_1 - s_1 + \alpha_A U + \alpha_S S) + \nu(1 - \zeta)(V - w_2 - s_2 + \alpha_A U + \alpha_S S) \\ & + (1 - \nu)(V - w_3 - s_3 + \alpha_A U + \alpha_S S) \end{aligned}$$

*s.t.*

$$\text{(AIR)} \quad w_i \geq 0$$

$$\text{(SIR)} \quad s_i \geq 0$$

$$\text{(AIC)} \quad w_2 \geq w_3 + \Delta \beta \hat{x}_3$$

$$(SIC) \quad s_1 - s_2 \geq k(w_2 - w_1)$$

Intuitively, S can get more in state 1 by telling truth than when he lies and gets a bribe  $k(w_2 - w_1)$ . The upper bound of P's revenue is  $\max\{\tilde{W}^d, \tilde{W}^u\}$ . The result is related to Theme 1 and can be referenced to Tirole (1986).

### Asymmetric information

Suppose that A doesn't know S's report. And S has two signals,  $\sigma = \phi$  and  $\sigma = \tilde{\nu}$ , where  $\tilde{\nu} > \nu$ . Consider the following mechanism: S has three potential reports  $r \in \{\sigma, \phi, \underline{\beta}\}$ .  $r = \underline{\beta}$  is a report of soft (unverifiable) information. S first reports  $r$  and then A announces  $\hat{\beta}$ . Rewards are as follows: S receives  $s = 0$  unless he reports  $r = \underline{\beta}$ . If  $r = \underline{\beta}$ ,  $s = k\Delta\beta$  if  $\hat{\beta} = \underline{\beta}$  ( $x = 1$  and  $w = \underline{\beta}$ ) and  $s = -\infty$  if  $\hat{\beta} = \bar{\beta}$  ( $x = 0$  and  $w = 0$ ). If  $r = \tilde{\nu}$ ,  $x = 1$  and  $w = \underline{\beta}$  if  $\hat{\beta} = \underline{\beta}$ , and  $x = w = 0$  if  $\hat{\beta} = \bar{\beta}$ . If  $r = \phi$ , then  $x = 1$  and  $w = \bar{\beta}$  for all  $\hat{\beta}$ .

Assume that A makes a take-it-or-leave-it offer  $t(r, \hat{\beta}, m)$  to S. We claim that *one* equilibrium is the no-collusion outcome, in which A offers the null side contract  $t(.,.,.) = 0$  and S and A report truthfully. Suppose that  $\sigma = \tilde{\nu}$ , any undominated offer by A is rejected by S, who can get  $k\Delta\beta$  by reporting  $r = \underline{\beta}$ . Of course, it is not interim efficient. Note that there exists another equilibrium of the collusion game in which A makes a take-it-or-leave-it offer and which yields the CIE (coalition-interim-efficient) outcome. When  $\sigma = \tilde{\nu}$  S reports  $\sigma = \phi$  and S and A share  $\Delta\beta$ .

So, when parties in a coalition are asymmetrically informed, there may be multiple equilibria of the coalition formation game. Augmented revelation mechanism (which builds on Nash implementation) may not be needed if one insists on strong collusion proofness.

### An application to China

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